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Equivalent physical models and formulation of equivalent source layer in high-resolution EEG imaging

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Abstract

In high-resolution EEG imaging, both equivalent dipole layer (EDL) and equivalent charge layer (ECL) assumed to be located just above the cortical surface have been proposed as high-resolution imaging modalities or as intermediate steps to estimate the epicortical potential. Presented here are the equivalent physical models of these two equivalent source layers (ESL) which show that the strength of EDL is proportional to the surface potential of the layer when the outside of the layer is filled with an insulator, and that the strength of ECL is the normal current of the layer when the outside is filled with a perfect conductor. Based on these equivalent physical models, closed solutions of ECL and EDL corresponding to a dipole enclosed by a spherical layer are given. These results provide the theoretical basis of ESL applications in high-resolution EEG mapping.

1. Introduction

High-resolution EEG imaging (HREI) is an intensively studied scientific area in recent years. The methods include the scalp Laplacian (SL) (Hjorth 1975, Babiloni *et al* 1996, He 1999, He *et al* 2001a, Yao 2002), cortical potential imaging (CPI) (Sidman *et al* 1992, Givins *et al* 1994, Yao 1996, Babiloni *et al* 1997, Wang and He 1998, He *et al* 1999, 2001b, 2002b) and equivalent source layers such as the equivalent dipole layer (EDL) (Sidman *et al* 1992, Dale and Sereno 1993, Yao 1995, He *et al* 2002a) and the equivalent charger layer (ECL) (Chernyak *et al* 1993, He *et al* 1995, Yao 1995, 1996). HREI is now being utilized in both clinical neurophysiological and psychological problems (Sidman *et al* 1992, He *et al* 2001b).

The rationale for ESL imaging is based on the fact that the scalp EEG is mainly generated by cortical sources. Therefore, by approximating cortical sources using a current dipole

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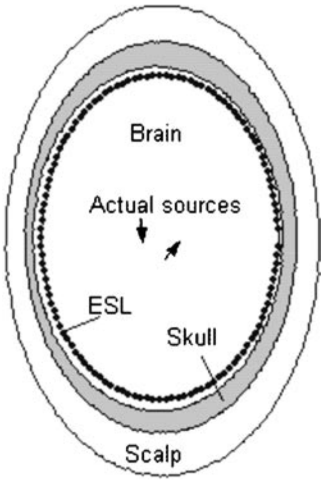

Figure 1. Illustration of the equivalent source layer model. A three layer head model is assumed, and the layer boundaries are plotted as disturbed non-concentric ellipses. The dotted line is the assumed equivalent source layer (ESL) which generates the same potential as the actual sources outside ESL including on the scalp surface.

layer normally oriented with respect to the local cortical surface (EDL), or by approximating cortical sources using a charge layer (ECL), one may obtain a fairly good approximation of brain electrical activity as observed from non-invasive scalp EEG. The basis is that the main pyramidal neurons assumed to be generating the scalp potential are parallel and normal to the cortical surface, and for each pyramidal cell, its macro performance is similar to a dipole, however, if we consider their top dendrites, they look like sinks that are similar to charges. If the scalp recordings are generated not only by cortical sources but also by neurons located in subcortical regions, both EDL and ECL provide an equivalent source representation of the underneath actual sources as shown below, because each of the EDL and ECL generates the same potential outside the layer as the actual sources. In the following sections, the focus is on the equivalent physical model of EDL and ECL, and it may be considered as a theoretical summary of the previous work on ECL and EDL (Sidman *et al* 1992, Yao 1995, 1996, 2000a, He *et al* 2002a, 2002b, Babiloni *et al* 1997).

2. ESL imaging technique

As shown in figure 1, both EDL and ECL consist of construction of an equivalent source (dipole or charge) layer (ESL) in a volume conductor which simulates the head so that the potentials generated by ESL would take the values measured on the scalp surface, i.e.

$$\sum_{i=0}^{N-1} u_i G(S_i, A_j) = \Phi(A_j) \qquad j = 1, \ldots, M. \tag{1}$$

where $\Phi(A_j)$ ($j = 1, \ldots, M$) are the electrical potentials on the body surface which are produced by actual neural electric sources, M is the number of electrodes, N is the number of discrete equivalent layer sources and u_i is the strength of a discrete equivalent source, $G(S_i, A_j)$ ($i = 0, \ldots, N-1$; $j = 1, \ldots, M$) is an element of the transfer matrix for a source located at S_i to an electrode at A_j, that is determined by the source type such as a dipole or a charge and the volume conductor model. The u_i may be obtained by singular value decomposition (SVD) (Sidman *et al* 1992) or other regularized inverse, then the potential outside the equivalent layer produced by ESL is assumed to be approximately the same as the actual potential produced by actual neural sources which are located inside the layer. In this way, ESL (EDL or ECL) can be utilized as an intermediate step to get the desired epicortical potential which is of higher spatial resolution than that of the scalp surface potential, and a theoretical solution of ESL based on a proper physical model will benefit by evaluating the efficiency of the inverse of ESL (Yao 2000a, He *et al* 2002a). Meanwhile, ESL can also be utilized as imaging modalities directly (He *et al* 2002a) and an equivalent physical model would be useful in guiding such applications.

In the following sections, the equivalent physical model of ESL and the closed solution of ESL for a dipole enclosed by a spherical layer are given.

3. Physical model of ESL

3.1. Equivalent source layer and integral solution of an electrostatic problem

Studies on biological tissues show that terms concerning permittivity and permeability in the governing equation of the bioelectricity over the EEG and ECG range of frequency can be neglected (Plonsey 1969). Accordingly, the head can be considered as a volume conductor having only the factor of conductivity and the governing equation formally becomes the same as that which is obtained under the static condition (Plonsey 1969, Gulrajani 1998).

The formal integral solution of the electrostatic boundary-value problem with the Green function is (Plonsey 1969, Gulrajani 1998)

$$\Phi(\vec{x}) = \int_v \rho(\vec{x}')G(\vec{x},\vec{x}')\,\mathrm{d}^3\vec{x}' + \oint_s \left[G(\vec{x},\vec{x}')\delta\frac{\partial\Phi(\vec{x}')}{\partial n'} - \delta\Phi(\vec{x}')\frac{\partial G(\vec{x},\vec{x}')}{\partial n'}\right]\mathrm{d}s' \qquad \text{for } \vec{x}' \text{ on } S. \tag{2}$$

The potential $\Phi(\vec{x})$ at $\vec{x}$ in the conductor region V which is the whole region except the source region V_0 bounded by the integral surface S, is composed of two terms, the first term is produced by a source with charge density $\rho(\vec{x})$ located in the region V, the second term is the contribution of sources in the source region V_0, the conductivity for region V is δ and for region V_0 is δ_0. Where $G(\vec{x},\vec{x}')$ is the Green function which is the potential solution at a field point $\vec{x}$ of a point source (charge) at $\vec{x}'$ in a volume conductor model, for example, for a unit charge in an infinite conductor, $G(\vec{x},\vec{x}') = \frac{1}{4\pi\delta|\vec{x}-\vec{x}'|}$ with $\vec{x}'$ on or outside the integral surface, and $\left(-\frac{\partial G}{\partial n'}\right)$ is the Green function of a dipole (Gulrajani 1998), S is the boundary surface between V and V_0 and $\frac{\partial}{\partial n'}$ is the normal derivative at the surface S (directed outside from V to V_0). For a neural electric problem, suppose all the actual sources are enclosed by the supposed ESL, for example, a layer located on the surface of the cortex which encloses all the neural electric sources inside, so the first term vanishes. Then we have

$$\Phi(\vec{x}) = \oint_s \left[G(\vec{x},\vec{x}')\delta\frac{\partial\Phi(\vec{x}')}{\partial n'} - \delta\Phi(\vec{x}')\frac{\partial G(\vec{x},\vec{x}')}{\partial n'}\right]\mathrm{d}s' \qquad \text{for} \quad \vec{x}' \text{ on } S. \tag{3}$$

Equation (3) shows that the contribution of sources in V_0 to the potential in V may be reproduced by a charge layer and a dipole layer, and for the charge layer, its strength density is $\delta\frac{\partial\Phi(\vec{x}')}{\partial n'}$ which is actually the normal current density on the surface, and for the dipole layer, its strength density is $\delta\Phi(\vec{x}')$ which is a quantity proportional to the surface potential $\Phi(\vec{x}')$. For convenience in the following discussions, we relabel $\Phi(\vec{x}')$ in the integral of equation (3) as $\Phi'(\vec{x}')$, then we have

$$\Phi(\vec{x}) = \oint_s \left[G(\vec{x}, \vec{x}')\delta \frac{\partial \Phi'(\vec{x}')}{\partial n'} - \delta \Phi'(\vec{x}') \frac{\partial G(\vec{x}, \vec{x}')}{\partial n'} \right] \mathrm{d}s' \qquad \text{for} \quad \vec{x}' \text{ on } S. \tag{4}$$

3.2. A pseudo-boundary in a homogeneous region

Equation (4) is valid for any electrostatic boundary, for example, it may be a physical boundary with a little different or big different conductivity, and even for a pseudo-boundary in a homogeneous region. Apparently, for this special case, the boundary is an assumed virtual boundary, so $\frac{\partial \Phi'(\vec{x}')}{\partial n'} = \frac{\partial \Phi(\vec{x}')}{\partial n'}$, $\Phi'(\vec{x}') = \Phi(\vec{x}')$, and both EDL and ECL are needed to represent the actual contribution of the inner sources to $\Phi(\vec{x})$. Based on this formula, equation (4), we may have an equivalent source layer technique or imaging method similar to that of EDL or ECL but involving both a charge layer and a dipole layer at the same spatial position (figure 1). To implement such a technique the discrete unknown variables will be double those of either the EDL or ECL cases, so much more computation is needed, and such a cumbersome technique has never been suggested in the literature.

3.3. A boundary with zero outside conductivity

According to equation (4), if we have

$$\frac{\partial \Phi'(\vec{x}')}{\partial n'} = 0 \tag{5}$$

then

$$\Phi(\vec{x}) = \oint_s (-\delta \Phi'(\vec{x}')) \frac{\partial G(\vec{x}, \vec{x}')}{\partial n'} \mathrm{d}s' \qquad \text{for} \quad \vec{x}' \text{ on } S. \tag{6}$$

Equations (5) and (6) show that if the normal current on the boundary is zero, then the general integral solution reduces to the continuous EDL form whose discrete form was given by equation (1). In other words, the strength density of EDL is proportional to the surface potential produced by the actual sources inside the surface with outside conductivity being zero (figure 2) that the boundary condition equation (5) is satisfied automatically. This fact provides the equivalent physical model (figure 2) of EDL, and it tells that an inverted EDL from scalp potential is just like a new surface potential when the outer layers are stripped (He *et al* 2002a).

3.4. A boundary with infinite outside conductivity

According to equation (4), if we have

$$\Phi'(\vec{x}') = 0 \tag{7}$$

then

$$\Phi(\vec{x}) = \oint_s G(\vec{x}, \vec{x}') \left(\delta \frac{\partial \Phi'(\vec{x}')}{\partial n'} \right) \mathrm{d}s' \qquad \text{for} \quad \vec{x}' \text{ on } S. \tag{8}$$

Equations (7) and (8) show that if the surface potential of the boundary is zero, then the general integral solution reduces to the continuous ECL form whose discrete counterpart was already shown in equation (1). The strength density of ECL is the normal surface current density produced by the actual sources inside the surface with outside conductivity being infinite such that the boundary condition equation (7) is satisfied automatically (figure 3). This fact provides the equivalent physical model of ECL, and means an inverted ECL from the scalp

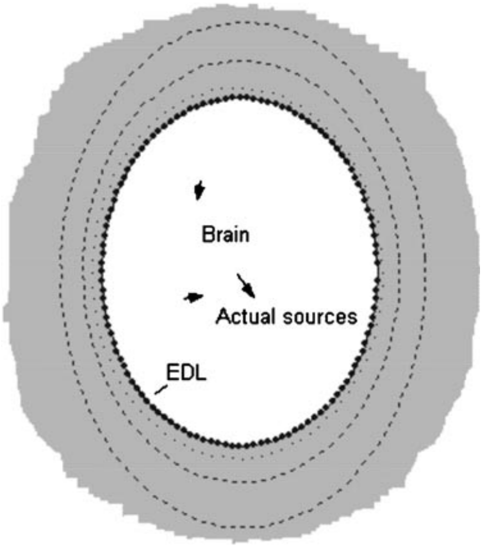

Figure 2. The equivalent physical model of the equivalent dipole layer. When the ESL in figure 1 is an EDL, the strength of the EDL is proportional to the surface potential of a physical model with the same actual sources and conductivity inside the layer (the white region) but with an absolute insulating medium outside (the grey region).

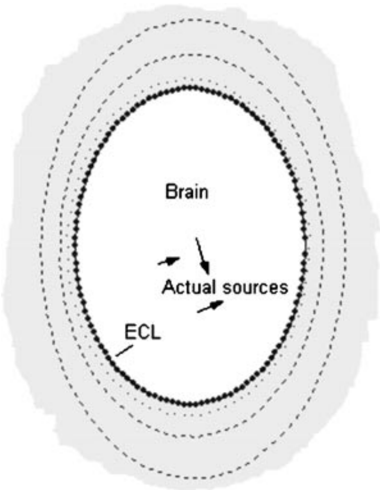

Figure 3. The equivalent physical model of the equivalent charge layer. When the ESL in figure 1 is an ECL, the strength of the ECL is equal to the normal current density on the surface of a physical model with the same actual sources and conductivity inside the layer (the white region) but with an absolute conducting medium outside (the grey region).

potential is just like the normal current density when the outside medium is replaced by a perfect conductor (figure 3).

Equations (6) and (8) are general formulations which mean that the forward EDL and ECL can be calculated by the boundary element method (BEM) or the finite element method (FEM) for an arbitrary volume conductor model and an arbitrary equivalent layer shape (Plonsey 1969, Chernyak *et al* 1993, Babiloni *et al* 1997, Gulrajani 1998, He *et al* 2002a). For example, for the EDL, we may get the EDL density, $-\delta\Phi'(\vec{x}')$, by solving a Poisson problem with the actual sources and the boundary condition equation (5) for an arbitrary boundary shape. This fact means that the ESL technique can be evaluated even for an arbitrary layered head model and an arbitrary equivalent source layer.

4. A closed solution of ESL of a spherical surface

Based on equations (7) and (8), for the ECL, we may also get the ECL density, $\delta\frac{\partial}{\partial n'}\Phi'(\vec{x}')$, by solving a Poisson problem with the actual sources and the boundary condition equation (7) for an arbitrary boundary shape. The general numeric solution of EDL and ECL for an arbitrary shape of the layer and arbitrary conductivity distribution of the volume conductor requires a numeric algorithm such as the finite element method (FEM) with much computation, while for a spherical model, such as multi-layer concentric and eccentric spheres and multi-layer spheroidal model, the existed forward series solutions of potential produced by a dipole are just the forward EDL (Gulrajani 1998), and by similar derivation, we may get the series solution of the forward ECL, too. Here we consider the simplest case, that the dipole is enclosed by a single spherical equivalent layer as used in the general EDL and ECL practice in EEG (Sidman *et al* 1992, Wang and He 1998, Yao 2003), which enables us to have the closed solutions and such closed solutions made the calculation of the theoretical EDL and ECL strength very easy.

4.1. Closed solution of a spherical EDL

As shown above, the equivalent physical model of EDL is a boundary between the volume conductor and the outside air, such as the head–air physical boundary. For a spherical surface and a dipole source inside, the closed solution has been derived in Yao (2000b).

4.2. Closed solution of a spherical ECL

In general, the potential produced by neural electric sources in an infinite medium may be assumed to be (Yao 2000a)

$$\Phi_0(r,\theta,\phi) = \sum_{l,m}\left(a_l^m \cos m\phi + b_l^m \sin m\phi\right)\frac{P_l^m(\cos\theta)}{r^{l+1}} \qquad \text{for} \quad r > r_0 \tag{9}$$

where P_l^m is the associated Legendre function of degree l and order m, a_l^m and b_l^m are the spherical harmonic spectra (SHS) of the potential, r_0 is the largest radius of the neural electric source positions, (r,θ,ϕ) are the spherical coordinates of a field point $\vec{x}$. According to the equivalent physical model and the uniqueness theory of boundary-value problem (Jackson 1975), we need to add to equation (9) a solution Φ_1 of Laplace's equation which has no poles in a region where $r < r_T$ and makes $\Phi' = \Phi_1 + \Phi_0$ satisfy the Dirichlet boundary condition $\Phi' = 0$ on S, equation (7). Then we may get the equivalent charge density strength by calculating the normal current density on the surface where the surface with radius r_T is the surface of the spherical layer. $\Phi_1(r,\theta,\phi)$ can be obtained by referencing to equation (9) and the Dirichlet boundary condition, and the total potential becomes

$$\begin{aligned}\Phi'(r,\theta,\phi) &= \Phi_1(r,\theta,\phi) + \Phi_0(r,\theta,\phi)\\ &= \sum_{l,m}(a_l^m \cos m\phi + b_l^m \sin m\phi)\left(\frac{1}{r^{l+1}} - \frac{r^l}{r_T^{2l+1}}\right)P_l^m(\cos\theta) \qquad r_T \geqslant r > r_0.\end{aligned} \tag{10}$$

Apparently, $\Phi'(r,\theta,\phi)|_s = \Phi'(r,\theta,\phi)|_{r=r_T} = 0.0$, and $\Phi_1(r,\theta,\phi)$is a solution of Laplace's equation in a region where $r < r_T$ and so $\Phi'(r,\theta,\phi)$ is the uniqueness solution in a region $r_T \geqslant r > r_0$ of the Poisson equation with a Dirichlet boundary condition on S $(r = r_T)$. According to equation (10), the strength of the spherical equivalent charge layer is

$$f_q = \delta \frac{\partial \Phi'}{\partial n}\bigg|_{r=r_T} = -\delta \frac{\partial \Phi'}{\partial r}\bigg|_{r=r_T} = \delta \sum_{l,m} Q \frac{1}{r_T^{l+1}} \left(a_l^m \cos m\phi + b_l^m \sin m\phi\right) P_l^m(\cos\theta) \tag{11}$$

with

$$Q = (2l+1)/r_T. \tag{12}$$

Equations (11) and (12) show that the charge layer strength density is a result of the original potential processed by the filter Q. This conclusion is the same as obtained in a direct comparison between the outside potential produced by actual sources and the potential produced by a spherical equivalent charge layer (Yao 2000a). Here in order to get a closed solution, we let

$$\begin{aligned} f_q &= \delta \sum_{l,m} \frac{2l+1}{r_T^{l+2}} \left(a_l^m \cos m\phi + b_l^m \sin m\phi\right) P_l^m(\cos\theta) \\ &= \delta \sum_{l,m} \frac{2(l+1)-1}{r_T^{l+2}} \left(a_l^m \cos m\phi + b_l^m \sin m\phi\right) P_l^m(\cos\theta) \\ &= \delta \sum_{l,m} \left(2\frac{l+1}{r_T^{l+2}} - \frac{1}{r_T^{l+2}}\right) \left(a_l^m \cos m\phi + b_l^m \sin m\phi\right) P_l^m(\cos\theta) \\ &= \delta \sum_{l,m} \left(2\left(-\frac{\partial}{\partial r}\left(\frac{1}{r^{l+1}}\right)\right)\bigg|_{r=r_T} - \frac{1}{r_T^{l+2}}\right) \\ &\quad \times \left(a_l^m \cos m\phi + b_l^m \sin m\phi\right) P_l^m(\cos\theta) \\ &= -2\delta \frac{\partial \Phi_0}{\partial r}\bigg|_{r=r_T} - \delta \frac{1}{r_T}\Phi_0\bigg|_{r=r_T} \end{aligned} \tag{13}$$

where Φ_0 is the potential produced by actual sources in an infinite homogeneous medium.

For a dipole in an infinite homogeneous medium, its corresponding Φ_0 is

$$\Phi_0|_{r=r_T} = \frac{1}{4\pi\delta} \frac{\vec{p}\cdot(\vec{r}_T - \vec{r}_0)}{|\vec{r}_T - \vec{r}_0|^3} \tag{14}$$

and so

$$\frac{\partial \Phi_0}{\partial r}\bigg|_{r=r_T} = \frac{1}{4\pi\delta}\left(\frac{\vec{p}\cdot\vec{r}_T}{r_T^4} - 3\frac{\vec{p}\cdot(\vec{r}_T - \vec{r}_0)}{r_T^5}\frac{r_T^2 - \vec{r}_0\cdot\vec{r}_T}{r_T}\right). \tag{15}$$

Invoking equations (14) and (15) into (13) we get the strength density of the equivalent charge layer of a dipole with moment $\vec{p}$ located at

$$\vec{r}_0 f_q = -\frac{1}{2\pi}\left(\frac{\vec{p}\cdot\vec{r}_T}{r_T^4} - 3\frac{\vec{p}\cdot(\vec{r}_T - \vec{r}_0)}{r_T^5}\frac{r_T^2 - \vec{r}_0\cdot\vec{r}_T}{r_T}\right) - \frac{1}{4\pi r_T}\frac{\vec{p}\cdot(\vec{r}_T - \vec{r}_0)}{|\vec{r}_T - \vec{r}_0|^3} \tag{16}$$

where vector $\vec{r}_0$ is the dipole location, vector $\vec{r}_T$ is a point on S. If r_T goes to infinity, the sphere decays to a plane, and formula (16) is simplified to contain only the first term, which was used in an equivalent body surface charge model (He *et al* 1995).

Equations (13)–(16) correspond to the case of a dipole in V_0. Similarly, we can derive a formula for a charge or a quadrupole source. This fact indicates that any source can be equivalently represented by a charge layer, and the physical basis is that any source model is formed by charge or charge combination (Plonsey 1969, Yao 1996, Gulrajani 1998).

5. Summary

In this paper, the equivalent physical model and forward formulation of the equivalent source layer utilized in current high-resolution EEG imaging are presented. The result of EDL indicates that it can be considered as the ‘potential’ on a new ‘scalp’ surface when the outer medium is replaced by an insulator, just like sometimes we get after the skull is open (He *et al* 2002a). While for ECL, its strength density is the normal current density, $j_r = -\delta \frac{\partial \Phi}{\partial r}\big|_{r=r_T}$, when the outer medium is replaced by a perfect conducting medium which guarantees a zero potential on boundary S. These results provide the theoretical base of the ESL application in practice, and the physical models provide a way to get the theoretical value of equivalent source layer even for an arbitrary layer shape.

About the equivalent source layer imaging, they are suggested not only because of their practical efficiencies as shown by He *et al* (2002a, 2002b) and Yao (2003), but also because of their clear physical meaning. The above-mentioned forward theory clearly affirms their being unique representations of the actual sources underneath the layer and being independent of the complexity of the outer medium of the actual head model (Yao 2003). Apparently, in this sense ECL and EDL are similar and prior to CPI and SL which are decided upon not only by the actual sources but also by the head model. Besides, if we approximately assume that the sources of our interest are on the cortical surface, the EDL and ECL may be considered as an approximate inversion of the actual sources.

The numeric results in He *et al* (2002a, 2002b) and Yao (2003) confirm that ECL and EDL are of higher spatial resolution than the scalp potential map and so of practical significance. In addition, EDL has been realized in a real head model (Babiloni *et al* 1997). ECL can be extended to a real head shape, too, so ECL and EDL can be expediently reconstructed from simulated data on a real head model or practical recordings. When comparing EDL and ECL, we see that ECL is of higher spatial resolution in theory because it is proportional to a derivative of the potential, see equation (8), and the strength of EDL is proportional to the potential directly, see equation (6); however, the practical inverse of each of them is an ill-posed problem, and due to their different ill-posedness that needs different regularization levels, the current available practical effects are similar to each other (Yao 2003).

Acknowledgments

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References

Babiloni F, Babiloni C, Carducci F, Fattorini L, Anello C, Onorati P and Urbano A 1997 High resolution EEG: a new model-dependent spatial deblurring method using a realistically-shaped MR-constructed subject’s head model *Electroenceph. Clin. Neurophysiol.* **102** 69–80

Babiloni F, Babiloni C, Carducci F, Fattorini L, Onorati P and Urbano A 1996 Spline Laplacian estimate of EEG potentials over a realistic magnetic resonance-constructed scalp surface model *Electroenceph. Clin. Neurophysiol.* **98** 363–73

Chernyak Y, He B and Cohen R J 1993 Theoretical basis of body surface charge mapping for real geometry torso *Proc. Ann. Int. Conf. of IEEE EMBS* vol 15 pp 756–7

Dale A M and Sereno M 1993 Improved localization of cortical activity by combining EEG and MEG with MRI cortical surface reconstruction: a linear approach *J. Cogn. Neurosci.* **5** 162–76

Givins A, Le J, Martin N and Reutter B 1994 High resolution EEG: 124-channel recording, spatial enhancement and MRI integration methods *Electroenceph. Clin. Neurophysiol.* **90** 337–58

Gulrajani R M 1998 *Bioelectricity and Biomagnetism* (New York: Wiley)

He B 1999 Brain electric source imaging: scalp Laplacian mapping and cortical imaging *Crit. Rev. Biomed. Eng.* **27** 149–88

He B, Chernyak Y B and Cohen R J 1995 An equivalent body surface charge model representative three-dimensional bioelectrical activity *IEEE Trans. Biomed. Eng.* **42** 637–46

He B, Lian J and Li G 2001a High-resolution EEG: a new realistic geometry spline Laplacian estimation technique *Clin. Neurophysiol.* **112** 845–52

He B, Lian J, Spencer K M, Dien J and Donchin E 2001b A cortical potential imaging analysis of the P300 and novelty P3 components *Hum. Brain Mapp.* **12** 120–30

He B, Wang Y and Wu D 1999 Estimating cortical potentials from scalp EEG's in a realistically shaped inhomogeneous head model by means of the boundary element method *IEEE Trans. Biomed. Eng.* **46** 1264–8

He B, Yao D and Lian J 2002a High-resolution EEG: on the cortical equivalent dipole layer imaging *Clin. Neurophysiol.* **113** 227–35

He B, Zhang Z, Lian J, Sasaki H, Wu S and Towle V L 2002b Boundary element method based cortical potential imaging of somatosensory evoked potentials using subjects' magnetic resonance images *Neuroimage* **16** 564–76

Hjorth B 1975 An online transformation of EEG scalp potentials into orthogonal source derivations *Electroenceph. Clin. Neurophysiol.* **39** 526–30

Jackson J D 1975 *Classical Electrodynamics* 2nd edn (New York: Wiley)

Plonsey R 1969 *Bioelectric Phenomena* (New York: McGraw-Hill)

Sidman R, Vincent D, Smith D and Lee L 1992 Experimental tests of the cortical imaging technique-applications to the response to median nerve stimulation and the localization of epileptiform discharges *IEEE Trans. Biomed. Eng.* **39** 437–44

Wang Y and He B 1998 A computer simulation study of cortical imaging from scalp potentials *IEEE Trans. Biomed. Eng.* **45** 724–35

Yao D 1995 Three dimensional imaging of EEG by equivalent source technique *J. Biomed. Eng.* **12** 332–41 (in Chinese)

Yao D 1996 The equivalent source technique and cortical imaging *Electroenceph. Clin. Neurophysiol.* **98** 478–83

Yao D 2000a High-resolution EEG mappings: a spherical harmonic spectra theory and simulation results *Clin. Neurophysiol.* **111** 81–92

Yao D 2000b Electric potential produced by a dipole in a homogeneous conducting sphere *IEEE Trans. Biomed. Eng.* **47** 964–6

Yao D 2002 High-resolution EEG mapping: a radial-basis function based approach to the scalp Laplacian estimate *Clin. Neurophysiol.* **113** 956–67

Yao D 2003 High-resolution EEG mapping: an equivalent charge layer approach *Phys. Med. Biol.* **48** 1997–2011